

# Generalized Formulae for Islamic Home Financing through the Musharakah Mutanaqisah Contracts

Abdullah Erođlu\*, řeref Kalaycı,  
Gültekin Özdemir, A. Cüneyt Çetin and  
Hayrettin Usul

## Abstract

*The origin of Musharakah is an Arabic term which signifies the sharing of an item. In the system of Musharakah, all parties share the profit or loss in a joint enterprise. It arose as an ideal alternative for interest-based financing systems for both the production and service sectors (Usmani, 1999). The model of Musharakah Mutanaqisah Partnership (MMP) is an interest-free financing model in which capital is not permanent and every repayment of capital by the entrepreneur will diminish the total capital ratio for the capital provider. When the capital ratio for the capital provider declines to zero, the entrepreneur becomes the sole owner for the business (Abdul Rahman, 2007). Generally, the MMP models are related to home financing and repayments are constant. In this study, general formulae are derived for the case in which repayments occur as a linear-gradient series for the MMP model. In addition, an illustrative example is presented.*

**Keywords:** *Musharakah, Mutanaqisah Partnership, Home Financing, Interest-free Mortgage.*

## Introduction

The Musharakah Mutanaqisah Partnership (MMP) model is an Islamic finance model based on a diminishing partnership concept (Meera

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\* (Corresponding Author) Süleyman Demirel University, Faculty of Economics and Administrative Sciences, Department of Management, 32260 Çünür, Isparta, Turkey.  
E-Mail: eroglu@ibbf.sdu.edu.tr

and Razak, 2005). The MMP model works as follows for home financing: A customer signs a contract with a financial institution to buy a home a certain percentage of which is financed by the customer and the remaining part of which is financed by the financial institution. The customer moves into the home and pays rent to the financial institution. The sharing of the rent occurs at the end of each month according to the proportions of each party. The customer also pays some amount of money in addition to his share of the rent to the financial institution each month to become in due course the 100% owner of the home. Therefore, the share of the customer increases with rental payments and repayments and reaches a figure of 100% some time after the application of the MMP model ends.

Greco, and Matthews et al. compared conventional mortgage models with the MMP model and concluded that the MMP model has some advantages over a conventional mortgage (scheme). These authors labeled the MMP model as an ‘Islamic mortgage’ or ‘share equity home financing model’.

The general formulae where repayments are constant are obtained by Meera and Razzak (2005). Numerical examples are given by Abidin et al., Hijazi and Hanif, Meera and Razzak (2009), Rammal (2004), and Siswanto and Qoyyimah. There would be other models with variable repayments too. Eroğlu et al. (2010) derived general formulae for the model with geometric gradient repayments.

It is important to reach more customers to provide for an increase for the repayment options in the MMP model. In this study, general formulae for the model with repayments as linear gradient series are derived and the model is supported with a numerical example.

The following notations are used in this study:

- $C_0$  buyer’s initial equity,
- $C_k$  buyer’s equity at the end of  $k^{\text{th}}$  month,
- $B_0$  financing institution’s (or coop bank’s) initial equity,
- $P$  purchasing price of the property, therefore,  $P = C_0 + B_0$ ,
- $A_k$  repayment to the financing institution at the end of  $k^{\text{th}}$  month,
- $A$  repayment to the financing institution at the end of  $1^{\text{th}}$  month, i.e.  $A = A_1$ ,
- $R$  monthly rental income of the home,
- $R_k$  rental income belonging to buyer at the end of  $k^{\text{th}}$  month, i.e.,  $R_k = R \left( \frac{C_{k-1}}{P} \right)$ ,
- $n$  number of repayments.

## Derivation of Generalized Formulae for the MMP Model with Repayments Occur as a Linear Gradient Series

The MMP model works as follows: A home with the purchasing price of  $P$  is bought while the amount of  $C_0$  is paid by the customer and the remaining amount ( $B_0$ ) is paid by the financing institution. The customer moves into the home and the share of the customer becomes  $C_0/P$ . The customer pays his/her rent to the financing institution at the end of each month. The customer's portion of the rent [ $R_k = R \left( \frac{C_{k-1}}{P} \right)$ ] is accepted by the financing institution as repayment at the end of each month. Thus, repayment from the rent increases with respect to the prior month. The customer can become the 100% owner of the home in a shorter time by paying some amount of money ( $D_k$ ) in addition to his share of the rent to the financial institution each month. In this case, the frequently-used repayment plan is one of repayments in constant amounts. In addition, repayments can be calculated as geometric or linear gradient series. Having more repayment options for financing institutions are very important in terms of reaching more customers. In this section, general formulae are derived for the case in which repayments occur as a linear-gradient series for the MMP model. The situation described by a linear gradient series of repayments could be defined as follows: Repayments occur as periods, e.g. months, and the difference between the repayments of sequential periods is constant. If the amount of change in repayments (increase or decrease) in a sequential period is defined with  $v$  then the formula of the amount of repayments by the customer can be written as follows:

$$A_k = A + (k-1)v, \quad k = 1, 2, \dots, n. \quad (1)$$

In other words, equation (1) forms the conditions of repayments for the linear gradient series.

Since, the buyer's equity at the end of  $k$ th month equals the summations of the buyer's equity at the end of  $(k-1)$ th month, the rental income owed by the buyer at the end of  $k$ th month and the repayment to be paid by the buyer at the end of  $k$ th month, can be written as follows:

$$\begin{aligned}
C_k &= C_{k-1} + R_k + A_k \\
&= C_{k-1} + \left(\frac{C_{k-1}}{P}\right)R + A_k \\
&= C_{k-1}F + A_k, \quad k = 1, 2, \dots, n
\end{aligned} \tag{2}$$

where.  $F = 1 + \frac{R}{P}$ .

The following expressions can be obtained from the equation (2) for  $k= 0, 1, 2, \dots, n$ :

$$C_0 = C_0$$

$$\begin{aligned}
C_1 &= C_0F + A_1 \\
&= C_0F + A
\end{aligned}$$

$$\begin{aligned}
C_2 &= C_1F + A_2 \\
&= C_0F^2 + AF + A + v \\
&= C_0F^2 + A(I + F) + v
\end{aligned}$$

$$\begin{aligned}
C_3 &= C_2F + A_3 \\
&= C_0F^3 + AF^2 + AF + vF + A + 2v \\
&= C_0F^3 + A(I + F + F^2) + v(I + F) + v
\end{aligned}$$

$$\begin{aligned}
C_4 &= C_3F + A_4 \\
C_4 &= C_0F^4 + A(F + F^2 + F^3) + v(F + F^2) + vF + A + 3v \\
C_4 &= C_0F^4 + A(I + F + F^2 + F^3) + v(I + F + F^2) + v(I + F) + v
\end{aligned}$$

$$\begin{aligned}
C_j &= C_0F^j + A\left(\sum_{k=0}^{j-1} F^k\right) + v\sum_{m=0}^{j-2} \sum_{k=0}^m F^k \quad (\text{see appendix}) \\
&= C_0F^j + \left(\frac{P}{R}\right)\left[A + \left(\frac{vP}{R}\right)\right](F^j - 1) - \frac{v j P}{R}, \quad j = 1, 2, \dots, n
\end{aligned} \tag{3}$$

Since the buyer's equity would be P at the end of nth month, the following equation is derived from equation (3):

$$C_n = P \text{ or } P = C_0 F^n + \left(\frac{P}{R}\right) \left(A + \frac{vP}{R}\right) (F^n - 1) - \frac{vnP}{R} \quad (4)$$

The following equations can be obtained from equation (4):

$$v = \frac{R[P(A+R) - (C_0 R + PA)F^n]}{P[P(F^n - 1) - nR]}, \quad (5)$$

$$C_0 = \frac{P - \left(\frac{P}{R}\right) \left[\left(A + \frac{vP}{R}\right) (F^n - 1) - vn\right]}{F^n}, \quad (6)$$

$$A = \frac{P\left(R + vn + \frac{vP}{R}\right) - F^n + \left(C_0 R \frac{vP^2}{R}\right)}{P(F^n - 1)} \quad (7)$$

### Example

A customer signs a contract with a bank and buys a home worth \$100,000 and moves into the home. Let's assume that the customer makes a down payment of \$ 20,000 and \$80,000 of the home would be financed by the bank. Thus, the customer owns 20% of the home and the bank owns 80% of the home. The rental income of the home is \$500 per month. The customer pays the rent to the bank at the end of each month. Both parties share the rental income according to their portion on the home. Therefore, the customer's proportion of the rental income is paid to the bank as repayment. Also, the customer pays some amount of money in addition to his share of the rent to the bank. If the customer wants to end the mortgage period within 120 months when monthly repayments increases by \$3, what would be the first monthly repayment ( $A=A_1$ )?

$$P = 100,000 \quad C_0 = 20,000, \quad B_0 = 80,000 \quad n = 120 \quad R = 500, \quad v = 3$$

$A$  is calculated as \$227.51 from equation (7). Table 1 summarizes the buyer's equity, rental share, repayments, and the financing institution's equity and rental income for 120 months.

**Table 1.** Solution Results for Repayments with a Linear Gradient Series

The Order of The Payment ( $k$ )	The Buyer				The Financing Institution (or coop bank)		
	Equity (\$) ( $C_k$ ) [from (3)]	Equity Percentage [( $C_k/P$ )*100]	Rental Share (\$) ( $R_k$ )	Repayment (\$) ( $A_k$ ) [from (1)]	Equity (\$) ( $P-C_k$ )	Equity Percentage	Rental Share (\$) ( $R-R_k$ )
	20,000.00	20.000	---	----	80,000.00	80.000	-----
0	20,327.51	20.328	100.00	227.51	79,672.49	79.672	400.00
1	20,659.66	20.660	101.64	230.51	79,340.34	79.340	398.36
2	20,996.47	20.996	103.30	233.51	79,003.53	79.004	396.70
3	21,337.96	21.338	104.98	236.51	78,662.04	78.662	395.02
4	-----	-----	-----	-----	-----	-----	-----
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25	29,633.89	29.634	145.94	299.51	70,366.11	70.366	354.06
26	30,084.57	30.085	148.17	302.51	69,915.43	69.915	351.83
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85	66,939.94	66.940	330.65	479.51	33,060.06	33.060	169.35
86	67,757.15	67.757	334.70	482.51	32,242.85	32.243	165.30
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118	-----	-----	-----	-----	-----	-----	-----
119	97,850.12	97.850	483.94	578.51	2,149.88	2.150	16.06
120	98,920.88	98.921	489.25	581.51	1,079.12	1.079	10.75
	100,000.00	100.000	494.60	584.51	0.00	0.000	5.40

If  $v=0$ , then equations (3), (6) and (7) reduce to equations (8),(9) and (10) respectively.

$$C_j = \left( C_0 + \frac{AP}{R} \right) F^j - \frac{AP}{R}, j = 1, 2, L, n \tag{8}$$

$$C_0 = \frac{P [R - A(F^n - I)]}{RF^n}, \tag{9}$$

$$A = \frac{PR - F^n C_0 R}{P(F^n - I)}. \tag{10}$$

The equations (8),(9) and (10) belong to the MMP model with constant repayments.

### Conclusion

Generally in Islamic home financing models, repayments are constant. Offering different repayment options for financing institutions allows access to a greater number of customers. Therefore, introducing new

finance models is extremely important. In this study, a new MMP model is presented where the repayments are different than the current financing models. The repayments of the presented model form a linear gradient series. General formulae are derived for the model and the model is supported with an example. This study constitutes a basis for new finance models for future research purposes. Different types of finance models help financing institutions to reach more diverse customer profiles and generate greater profit, so providing an opportunity to the general economy to realize increased turn over.

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## APPENDIX

$$\begin{aligned}
 \sum_{m=0}^{j-2} \sum_{k=0}^m F^k &= \sum_{m=0}^{j-2} \left( \frac{F^{m+1} - 1}{F - 1} \right) \\
 &= \left( \frac{1}{F - 1} \sum_{m=0}^{j-2} F^{m+1} \right) + \frac{1}{F - 1} \sum_{m=0}^{j-2} (-1) \\
 &= \left( \frac{F}{F - 1} \right) \left( \frac{F^{j-1} - 1}{F - 1} \right) + \frac{1 - j}{F - 1} \\
 &= \left( \frac{1}{F - 1} \right) \left[ \frac{F(F^{j-1} - 1)}{F - 1} + 1 - j \right] \\
 &= \left( \frac{P}{R^2} \right) [PF(F^{j-1} - 1) + R(1 - j)]
 \end{aligned}$$